

Name: _____

Date: _____

Solving Quadratic Word Problems II

Algebra 1 Class work / Homework

Today, we continue our work with quadratic word problems by solving several problems with similar contexts as yesterday. All problems **must** be solved *algebraically*. As always, follow the guidelines below in doing all word problems:

STEPS IN SOLVING WORD PROBLEMS WITH ALGEBRA

1. Define the variables that you want to find with let statements.
2. Create equation(s) that express the information given in the problem's scenario.
3. Solve using algebraic methods.
4. Consider if your answer(s) is/are reasonable.
5. Label your solution(s) appropriately.
6. Check your answer(s) with the conditions given in the problem.

1. The product of two consecutive integers is 30. Find the integers.

$$\begin{array}{l}
 x \\
 x+1
 \end{array}
 \quad
 \begin{array}{l}
 x(x+1) = 30 \\
 x^2 + x = 30 \\
 x(x+1) = 0 \\
 x=0 \quad x+1=0 \\
 x = -1
 \end{array}
 \quad
 \begin{array}{l}
 x^2 + x - 30 = 0 \\
 (x+6)(x-5) = 0 \\
 x+6=0 \quad x-5=0 \\
 x = -6 \quad x = 5
 \end{array}
 \quad
 \begin{array}{l}
 (6, 5) \\
 \text{check} \\
 6 \cdot 5 = 30 \\
 (-6, -5)
 \end{array}$$

2. Find three consecutive positive odd integers such that the product of the first and the third is 4 less than 7 times the second.

$$\begin{array}{l}
 x \\
 x+2 \\
 x+4
 \end{array}
 \quad
 \begin{array}{l}
 x(x+4) = 7(x+2) - 4 \\
 x^2 + 4x = 7x + 14 - 4 \\
 x^2 + 4x = 7x + 10 \\
 x^2 - 3x - 10 = 0 \\
 (x-5)(x+2) = 0 \\
 x-5=0 \\
 x = 5
 \end{array}
 \quad
 \begin{array}{l}
 2 \\
 (5, 7, 9)
 \end{array}$$

3. An object is moving in a straight line. It initially travels at a speed of 6 meters per second, and it speeds up at a constant acceleration of 4 meters per second each second. The distance d , in meters, that this object travels is given by the equation $d = 2t^2 + 6t$, where t is in seconds. According to this equation, how long will it take the object to travel 108 meters?

$$108 = 2t^2 + 6t$$

$$2t^2 + 6t - 108 = 0$$

$$t^2 + 3t - 54 = 0$$

$$(t+9)(t-6) = 0$$

$$t = 6 = 0$$

$$t = 6 \text{ seconds}$$

4. The entrance to an athletic field is in the shape of a parabolic archway. The archway is modeled by the equation $d = 12x - x^2$, where d represents the distance, in feet, that the arch is above the ground for any x value.

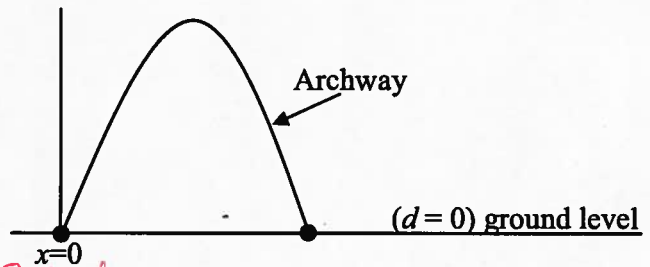
- (a) For what values of x will the arch be 20 feet above the ground?

$$20 = 12x - x^2$$

$$x^2 - 12x + 20 = 0$$

$$(x-10)(x-2) = 0$$

2 feet to 10 feet



- (b) How many feet wide is the base of the arch?

$$12x - x^2 = 0$$

$$x(12 - x) = 0$$

$$x = 0 \quad 12 - x = 0$$

$$x = 12$$

12 feet

- (c) What is the maximum height of the arch above the ground?

$$-\frac{b}{2a} = \frac{12}{2(-1)} = 6$$

$$12(6) - 6^2 = 36 \text{ inches}$$

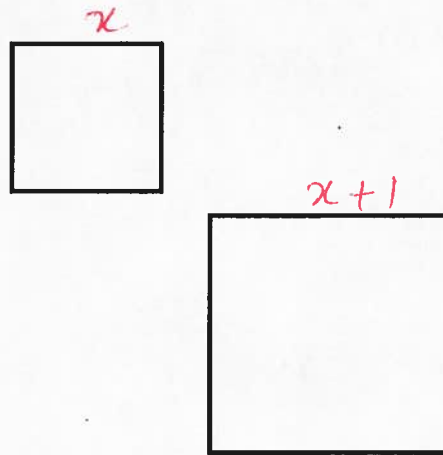
5. The product of two consecutive negative even integers is 24. Find the integers.

$$\begin{aligned}
 x & \\
 x+2 & \\
 x(x+2) &= 24 \\
 x^2 + 2x - 24 &= 0 \\
 (x+8)(x-6) &= 0 \\
 x+8 &= 0 \\
 x &= -8
 \end{aligned}$$

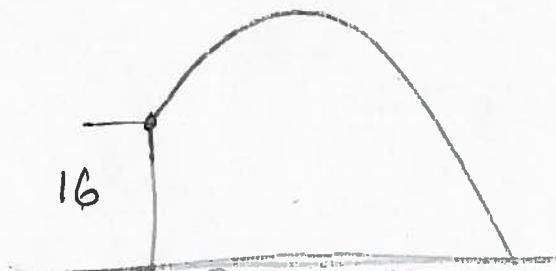
$$-6, -4$$

6. In the two squares shown below, the longer square has a side length 1 foot greater than that of the smaller square. If the combined area of the two squares is 113 square feet, find the length of the side of the smaller square. (Define any variables that you use by using let statements and/or by labeling the diagram.)

$$\begin{aligned}
 x^2 + (x+1)^2 &= 113 \\
 x^2 + x^2 + 2x + 1 &= 113 \\
 2x^2 + 2x - 112 &= 0 \\
 (x + \frac{16}{2})(x - \frac{14}{2}) &= 0 \\
 (x+8)(x-7) &= 0 \\
 x &= 7
 \end{aligned}$$



7. An object is launched into the air from a ledge 16 feet off the ground at an initial vertical velocity of 96 feet per second. Its height H , in feet, at t seconds is given by the equation $H = -16t^2 + 96t + 16$. Find all times t that the object is at a height of 160 feet off the ground.



$$160 = -16t^2 + 96t + 16$$

$$160 + 16t^2 - 96t - 16 = 0$$

$$\frac{16t^2 - 96t - 144}{16} = 0$$

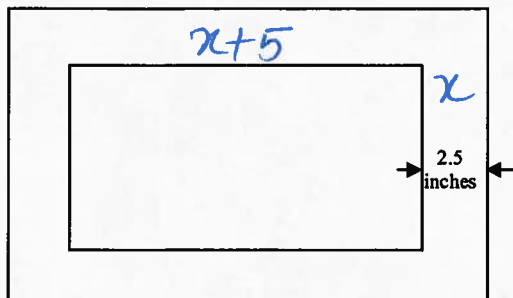
$$t^2 - 6t - 9 = 0$$

$$(t-3)(t+3) = 0$$

$$\begin{aligned}
 t-3 &= 0 & t+3 &= 0 \\
 t &= 3 & & -3
 \end{aligned}$$

$$t = 3$$

8. As illustrated below, a frame for a picture is $2\frac{1}{2}$ inches wide. The picture enclosed by the frame is 5 inches longer than it is wide. If the area of the picture itself is 300 square inches, determine the outer dimensions of the frame.



$$\begin{aligned}x(x+5) &= 300 \\x^2 + 5x &= 300 \\x^2 + 5x - 300 &= 0 \\(x+20)(x-15) &= 0 \\x-20 &= 0 \\x &= 20\end{aligned}$$

$$(20 + 2.5)(25 + 2.5)$$

width \uparrow 25 in
length \downarrow 20 in

9. The square of a positive number increased by 4 times the number is equal to 140. Find the number.

$$\begin{aligned}x^2 + 4x &= 140 \\x^2 + 4x - 140 &= 0 \\(x+14)(x-10) &= 0 \\x-10 &= 0 \\x &= 10\end{aligned}$$

$$100 + 40 = 140 \checkmark$$

10. Find three consecutive positive odd integers such that the product of the first and third is equal to 1 less than twice the second.

$$\begin{aligned}x \\x+2 \\x+4\end{aligned}$$

$$\begin{aligned}x(x+4) &= 2(x+2) - 1 \\x^2 + 4x &= 2x + 4 - 1 \\x^2 + 2x - 3 &= 0 \\(x+3)(x-1) &= 0 \\x-1 &= 0 \\x &= 1\end{aligned}$$

$$\begin{aligned}1, 3, 5 \\1(5) &= 2(3) - 1 \\5 &= 6 - 1\end{aligned}$$